

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 3503

FINAL EXAMINATION

APRIL 2000

TIME: 3 HOURS

NO CALCULATORS PERMITTED

MARKS

- (3) 1. Find the Laplace transform of

$$f(t) = \begin{cases} 2t + 1 & , \quad 0 \leq t < 2 \\ 3t & , \quad t \geq 2 \end{cases} .$$

- (3) 2. Find $\mathcal{L}\{\cosh t \sin 2t\}$.

- (2) 3. Find $\mathcal{L}^{-1}\left\{\frac{3s + 8}{s^2 + 2s + 5}\right\}$.

- (6) 4. Use Laplace transforms to solve

$$y'' + 6y' + 5y = 3e^{-2t} + 2\delta(t - 1), \quad y(0) = -3, \quad y'(0) = 2.$$

- (3) 5. Solve the Euler equation

$$6x^2y'' + 5xy' - y = 0.$$

- (6) 6. Use the Method of Frobenius to find one solution to

$$x^2y'' + 2xy' - x^2y = 0.$$

- (6) 7. Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

- (7) 8. Solve the system

$$\vec{x}' = \begin{bmatrix} 3 & 4 & -10 \\ 2 & 1 & -2 \\ 2 & 2 & -5 \end{bmatrix} \vec{x}$$

given that the characteristic polynomial is $-(\lambda - 1)(\lambda + 1)^2$.

(6) 9. Solve the system

$$\vec{x}' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \vec{x} + \begin{bmatrix} 2e^{4t} \\ e^{4t} \end{bmatrix}.$$

(5) 10. Find the Fourier series

$$f(x) = \begin{cases} 1 & , \quad -5 < x < 0 \\ 1+x & , \quad 0 \leq x < 5 \end{cases} \quad , \quad f(x+10) = f(x) \quad .$$

(6) 11. For the function

$$f(x) = \cos x, \quad 0 < x < \pi$$

find (a) the Fourier cosine series and (b) the Fourier Sine series and sketch the graphs of the functions to which each series converges.

(7) 12. Use separation of variables to solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$u_x(0, y) = u_x(\ell, y) = 0, \quad y \geq 0$$

$$u(x, 0) = f(x), \quad u_y(x, 0) = 0, \quad 0 \leq x \leq \ell.$$

(60)