DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 3503

FINAL EXAMINATION

December, 2006 Time: 3 HOURS

NO CALCULATORS

MARKS

(7) 1. (a) Use undetermined coefficients to find the general solution to the differential equation

$$y'' + +2y' + 5y = 2\sin x.$$

(7) (b) Use variation of parameters to solve

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 2e^{-3x}.$$

2. Find the Laplace transform of the following functions:

(3) (a) $f(t) = t^{3/2}e^{-2t}$ (answer should contain $\sqrt{\pi}$)

(4) (b) $f(t) = \begin{cases} t^2, & \text{if } 0 < t < 2 \\ 0, & \text{if } t > 2 \end{cases}$

(3) 3. (a) Find $\mathcal{L}^{-1}\left\{\dfrac{s}{(s^2+3)^2}e^{-2s}\right\}$.

(4) (b) Use the convolution theorem to find

$$\mathcal{L}^{-1}\left\{\frac{1}{(s^2+2s+2)(s^2+4)}\right\}.$$

Do not evaluate the convolution integral.

(7) 4. Use the Laplace transform to solve the initial value problems.

$$y'' + 4y' + 13y = \delta(t-3), \quad y(0) = 1, \quad y'(0) = 0.$$

(7) 5. Solve

$$\frac{d^2x}{dt^2} + \omega^2 x = f(t), \quad x(0) = x_0, \quad x'(0) = x_1,$$

where ω, x_0, x_1 are constants and $f(t)$ is an arbitrary function. Part of your solution should be expressed as a convolution integral.

(7) 6. Find the general solution to the system in terms of real functions

$$\frac{dx}{dt} = 3x - 2y$$

$$\frac{dy}{dt} = 5x + y$$

(7) 7. Find the general solution to the system of differential equations

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}, \quad \text{where} \quad A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix},$$

given that the characteristic equation for the coefficient matrix A is

$$-(\lambda + 1)(\lambda^2 - 7\lambda - 8) = 0.$$

(7) 8. Find the Fourier cosine series for the function

$$f(x) = x, \quad 0 < x < 1.$$

Sketch the graph of the function to which the series converges for $-3 \leq x \leq 3$.

(7) 9. Find Fourier series for

$$f(t) = \begin{cases} 0, & \text{if } -2 < t < 0 \\ 1, & \text{if } 0 < t < 2 \end{cases}$$

Sketch the graph of the function to which the series converges for $-6 \leq t \leq 6$.

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