

DEPARTMENT OF MATHEMATICS & STATISTICS

MATH 3803

FINAL EXAMINATION  
DECEMBER 1999

TIME: 3 HOURS  
TOTAL: 60

WORK MUST BE DONE IN PEN.

PART A

Please attempt ALL the questions. Calculators must not be used.  
Each question is worth 6 marks.

1. If  $A(t) = e^{0.01(1+t+t^2)}$ , find
  - (a)  $a(t)$
  - (b)  $I_4$
  - (c)  $i_4$
  - (d)  $\delta_t$
  - (e)  $\delta_4$
  
2. Find  $\ddot{a}_{\overline{10}|}$ ,  $5|\ddot{a}_{\overline{10}|}$ ,  $5|a_{\overline{10}|}$ ,  $\ddot{s}_{\overline{10}|}$ ,  $1|a_{\overline{10}|}$  where  $i = 0.05$ .
  
3. Jean-Pierre must repay a loan of \$10,000 over 4 years by payments made at the end of each month. If interest is 12% compounded monthly, what will be his monthly payments?
  
4. Money is to be collected by the friends of the CIE to endow a scholarship. The scholarship will have a value of \$500 per annum in perpetuity.
  - (a) If interest is at 5% per annum and the scholarship is paid out at the end of each year, what must be the size of the endowment?
  - (b) The money for the endowment is to be collected by the friends and deposited at the end of each year for 20 years. If interest is 5% per annum, what must be the size of the annual deposits to achieve the required endowment value?

## PART B

Please attempt ALL the questions. Calculators may be used.

5. Anne-Marie borrows \$5,000 from the Neighbourhood Finance Company at the rate of 12% per annum convertible quarterly. After 2 years she repays \$3,000 and after 3 years she repays \$2,000. How much does she still owe 5 years after the loan was taken out?
  
6. A loan of \$10,000 on January 1<sup>st</sup>, 1998 is to be repaid by semi-annual payments of \$800 with the first payment made on July 1<sup>st</sup>, 1998. Find the date of the last payment, and its value, if the last payment is larger than the regular payments. Assume  $i^{(2)} = 8\%$ .
  
7. Annabel makes semi-annual deposits of \$400 into a bank savings account. After 8 years, she finds that when she has just made her deposit the account contains \$8,100.00.
  - (a) What is the nominal rate of interest compounded semi-annually? (Use linear interpolation.)
  - (b) What is the equivalent effective annual rate of interest?
  
8.
  - (a) Give the first four lines of an amortization schedule for a loan of \$100,000 with monthly payments over 15 years where  $i^{(12)} = 12\%$ .
  - (b) Give the first three lines of the corresponding sinking fund schedule where the interest rate on both the loan and sinking fund is  $i^{(12)} = 12\%$ .

## PART C

**Please attempt TWO questions.**

9. (a) For the mortgage in question 8 (a), what is the outstanding loan balance after 10 years?
- (b) How much of the first payment in year 11 goes to repaying the principal?
- (c) What is the total amount of interest paid over the whole life of the loan?
10. Consider the mortgage in question 8 (a).
- (a) If instead the loan is amortized over 25 years, what are the revised monthly payments?
- (b) If the difference between payments in question 8 (a) and payments in question 10 (a) is invested monthly in investments with an interest rate of  $i^{(12)} = 15\%$ , how much will be accumulated in investments at the end of 15 years?
- (c) What is the outstanding loan balance (mortgage) after 15 years?
- (d) If the investments in 10 (b) is used to pay off the mortgage in 10 (c) at the end of the 15 years, how much money will be left in the investments?
11. Kelly has borrowed \$1,000 on which she is paying interest at  $11\frac{1}{2}\%$  effective per year. She is accumulating a sinking fund at 9% effective to repay the loan. At the end of the eighth year, the borrower makes a total payment of \$149.06.
- (a) How much of the \$149.06 pays interest on the loan?
- (b) How much of the \$149.06 goes into the sinking fund?
- (c) What is the sinking fund balance at the end of the eighth year?
- (d) In which year will the principal be paid off?
12. A mortgage of \$100,000 is to be amortized over 20 years by monthly payments. Interest is compounded semi-annually;  $i^{(2)} = 8\%$ .
- (a) What are the monthly payments?
- (b) What is the outstanding loan balance after 10 years?