UNIVERSITY OF NEW BRUNSWICK
DEPARTMENT OF MATHEMATICS & STATISTICS

MATHEMATICS 3503: Differential equations for engineers (Winter 2010)
Assignment 4 (due: Wednesday February 3, 2010)

1. A spring with a 1 kg mass attached to it has spring constant 18 N/m. There is damping with damping constant $c = 6$ kg/s and no external force. The mass is set in motion from the equilibrium position with a velocity of 2 m/s. Find the position $x = x(t)$ of the mass at any time t, where x is measured from the equilibrium position.

2. A spring with a 1 kg mass attached to it has spring constant 16 N/m. There is no damping and there is an external force given by $F(t) = 8\sin 4t$ N, where t is time. The mass is initially stretched 1 m beyond the equilibrium position and released. Find the position $x = x(t)$ of the mass measured from the equilibrium position.

3. Use the definition of the Laplace transform to find $\mathcal{L}\{f(t)\}$ where

$$f(t) = \begin{cases} e^{-3t}, & 0 \le t \le 3 \\ 3, & t > 3 \end{cases}$$

4. With the aid of tables, find the Laplace transform of the given function.

   (a) $f(t) = t^{\frac{5}{2}}$, answer should contain $\sqrt{\pi}$.

   (b) $f(t) = t^{\frac{2}{5}}$.

   (c) $f(t) = t^3 + 2t^2 - 3t - 1$.

   (d) $f(t) = 2\sin 2t - \cos 4t$.

   (e) $f(t) = 4\cos 2t \sin 2t$.