Ass 10

TOTAL = 17

(1)(a) Just compute

$$A\vec{v} = \begin{bmatrix} -5 & -8 & 2 \\ 3 & 6 & -2 \\ 4 & 4 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \\ -2 \\ -2 \end{bmatrix} = -2\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

2 Eigenvalue $= -2$

Parts (b),(c) - nex page; 5 more points

2.

$$D = PAP^{-1} = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 8 & 18 \\ -3 & -7 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 6 & 2 \\ -2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \text{ (diagonal)}$$

Thus

$$D^2 = \begin{bmatrix} 2^2 & 0 \\ 0 & (-1)^2 \end{bmatrix}, \quad D^3 = \begin{bmatrix} 2^3 & 0 \\ 0 & (-1)^3 \end{bmatrix}$$

etc

and $A = P^{-1}DP$, so

$$A^n = \underbrace{(P^{-1}DP)(P^{-1}DP)\cdots(P^{-1}DP)}_{n \text{ repeats}}$$

$$= P^{-1}\overbrace{D\cdot D \cdots D}\,P$$

$$= P^{-1}D^nP$$

so

$$A^{100} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 2^{100} & 0 \\ 0 & +1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3(2^{100})-2 & 3(2^{101})-6 \\ 1-2^{100} & 3-2^{101} \end{bmatrix}$$

2

3

(a) $zw = (2+i)(1-i) = 3-i$

2

(b) $\dfrac{z}{w} = \dfrac{2+i}{(1-i)}\cdot\dfrac{(1+i)}{(1+i)} = \dfrac{1+3i}{2}$

$= \dfrac{1}{2} + \dfrac{3}{2}i$ 2

(c) $\overline{z-3w} = \overline{(2+i)-(3-3i)}$

$= \overline{-1+4i} = -1-4i$

2

(d) $-3w = -3+3i$

$|-3w| = \sqrt{(-3)^2+3^2}$

$= \sqrt{18}$ OR $3\sqrt{2}$

2

(b) For $\lambda = 3$ solve homog. system

$(A - \lambda I)\vec{x} = \vec{0}$, ie $(A - 3I)\vec{x} = \vec{0}$

so reduce $[A - 3I \mid 0]$, ie

$$\left[\begin{array}{ccc|c} -8 & -8 & 2 & 0 \\ 3 & 3 & -2 & 0 \\ 4 & 4 & -1 & 0 \end{array}\right]$$

$$\downarrow$$

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \quad (x_2 \text{ free!})$$

so the eigenspace consists of all vectors

$$\vec{x} = t \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad t \in \mathbb{R}$$

2

(A typical eigenvector is $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$)

1

(any non-zero multiple will do.)

(c) The characteristic equation is

$$0 = \det(A - \lambda I) = \begin{vmatrix} (-5-\lambda) & -8 & 2 \\ 3 & (6-\lambda) & -2 \\ 4 & 4 & (2-\lambda) \end{vmatrix}$$

(expand say row 1)

$$= (-5-\lambda)\big((6-\lambda)(2-\lambda) + 8\big)$$
$$+ 8\,(3(2-\lambda) + 8)$$
$$+ 2\,(12 - 4(6-\lambda))$$

so

$$0 = -(\lambda+5)(\lambda^2 - 8\lambda + 20)$$
$$+ 8\,(14 - 3\lambda)$$
$$+ 2\,(4\lambda - 12)$$

$$= -[\lambda^3 - 3\lambda^2 - 4\lambda + 12]$$

But we know $\lambda = 3$ is a root (part (b)) so $\lambda - 3$ is a factor.

So divide:

$$\begin{array}{r} \lambda^2 \quad\;\; - 4 \\ \lambda - 3 \,\big)\, \overline{\lambda^3 - 3\lambda^2 - 4\lambda + 12} \\ \underline{\lambda^3 - 3\lambda^2 \qquad\qquad} \\ 0 - 4\lambda + 12 \\ \underline{-4\lambda + 12} \\ 0 \end{array}$$

so

$$0 = (\lambda - 3)(\lambda^2 - 4)$$

The eigenvalues are
3, 2, −2

ie Third eigenvalue = +2

2