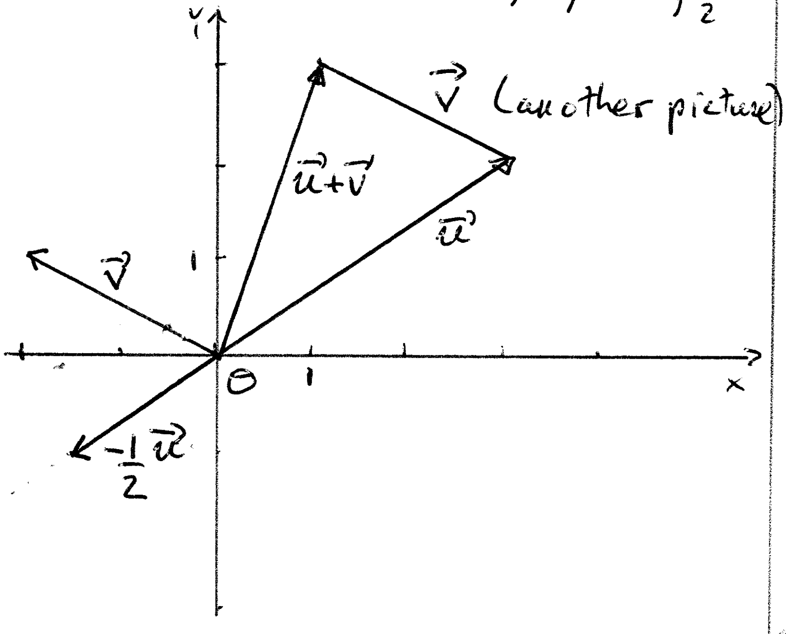
Ass. 2 - Total 8

(Bonus: $\frac{10}{8}$ possible!)

1. (a), (b), (c) - show $\overset{1}{\vec{u}, \vec{v}}$, $\overset{1}{\vec{u}+\vec{v}}$, $\overset{1}{-\frac{1}{2}\vec{u}}$



(d) One can do this geometrically but algebra is easier:

$$\begin{bmatrix}2\\1\end{bmatrix} = a\overset{\vec{u}}{\begin{bmatrix}3\\2\end{bmatrix}} + b\overset{\vec{v}}{\begin{bmatrix}-2\\1\end{bmatrix}}, \text{ so}$$

$$\begin{cases} 3a-2b=2 \\ 2a+b=1 \end{cases}$$ sub. last into first.

$b = 1-2a$

$3a - 2(1-2a) = 2$

$7a = 4$

so $\underline{a = \frac{4}{7}, \quad b = -\frac{1}{7}}$ (1)

(e) We need that (1)

$\underline{\vec{u}, \vec{v} \text{ are not parallel.}}$

(or: linearly independent)

(2) $a\begin{bmatrix}1\\-2\\3\end{bmatrix} + b\begin{bmatrix}2\\-4\\c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

so $\left.\begin{matrix} a+2b=0 \\ -2a-4b=0 \end{matrix}\right\}$ so $a=-2b$

$\downarrow$

$3a + bc = 0$, so $3(-2b)+bc=0$

so $b(c-6)=0$. But you can't use $b=0$ by request, so $c=6$.

(2) $\underline{\text{Answer}}$ $c = 6$; b can be any real except 0,

eg $b = 1$ (or -5, 10, etc)

so $\underline{\underline{a = -2}}$

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(3) $\underline{\text{Bonus}}$ - (2)

$\cos\theta = \dfrac{p \cdot q}{\|p\|\|q\|} = \dfrac{-2}{\sqrt{4}\sqrt{4}} = -\frac{1}{2}$,

so $\underline{\theta = 120^\circ}$ (exactly)

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(4) $\|\vec{w}\| = \sqrt{2^2+(-2)^2+(1)^2} = \sqrt{9} = 3$

so unit vector

(1) $= \frac{1}{3}\vec{w} = \frac{2}{3}\vec{i} - \frac{2}{3}\vec{j} + \frac{1}{3}\vec{k}$

OR $= \begin{bmatrix}2/3\\-2/3\\1/3\end{bmatrix}$ is OK.