Ass 6

TOTAL $= 16$

1 (a) $A^{509} = (A^2)^{254} \cdot A^1 = I^{254} A = I \cdot I \cdot \ldots \cdot I A = \underline{A}$ details not really needed.

(2)

(b) $B^2 = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}$ $B^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$B^4 = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$ $B^5 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$

$B^6 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

(2) B has period 6.

(2) Determinant $= \cos^2\theta - (-\sin^2\theta) = \cos^2\theta + \sin^2\theta = 1$

so

$$R^{-1} = \frac{1}{1}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(2)

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Extra: For a "rotation matrix" like this, $R^{-1} = R^T$.

Answer: The lines do intersect at $(4, 3, -2)$

3 (a) line dir^n vector $= \langle -2, 1, 1\rangle$ serves double duty as plane normal, so we get

$\underline{-2x + y + z = -1}$ (2)

(b) $\underline{x - 2y + z = -1}$ (2)

4. For $\tilde{x} = \frac{1}{x}$, $\tilde{y} = \frac{1}{y}$ we have $\begin{cases} \tilde{x} - 2\tilde{y} = 5 \\ 2\tilde{x} - 5\tilde{y} = 4 \end{cases}$. Solve any way for $\tilde{x} = 17$, $\tilde{y} = 6$

so

$x = \frac{1}{17}$, $y = \frac{1}{6}$ (2)

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(a) $L_1: \begin{cases} x = 1 + t \\ y = 0 + t \\ z = 1 - t \end{cases}$ (1)

(b) $L_2: \begin{cases} x = -1 + 10u \\ y = 0 + 6u \\ z = 0 - 4u \end{cases}$ (1)

(c) Lines intersect if above we can find t, u making same x, y, z

so $\left.\begin{array}{l} 1 + t = -1 + 10u \\ t = 6u \\ 1 - t = -4u \end{array}\right\}$ subst. $t = 6u$ into 1st, 3rd equations

(2) pts.

giving $2 = 4u$ and $1 = 2u$, so yes $u = \frac{1}{2}$ and $t = 3$