Ass 7
TOTAL = 22

1(a) Reduce

$$\left[\begin{array}{ccc|c} 2 & -1 & 1 & 7 \\ 1 & 1 & 2 & 5 \\ 1 & -2 & -3 & -4 \end{array}\right]$$

$R_1 \leftrightarrow R_2$ then new $R_2 - 2R_1$, $R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & -3 & -3 & -3 \\ 0 & -3 & -5 & -9 \end{array}\right]$$

$-\frac{1}{3} R_2$ then $R_3 + 3R_2$, $R_1 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -2 & -6 \end{array}\right]$$

$-\frac{1}{2} R_3$ then $R_1 - R_3$, $R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Unique solution: $\begin{cases} x = 1 \\ y = -2 \\ z = 3 \end{cases}$ (4)

[solution must be clearly stated]

Note: ERO's could be done differently. But the end result will be the same

(b)

$$\left[\begin{array}{cccc|c} 1 & 2 & -1 & 3 & 5 \\ 2 & 4 & 0 & 1 & 6 \end{array}\right]$$

$R_2 - 2R_1$

$$\left[\begin{array}{ccccc} 1 & 2 & -1 & 3 & 5 \\ 0 & 0 & 2 & -5 & -4 \end{array}\right]$$

$\frac{1}{2} R_2$, $R_1 + \text{new } R_2$

$$\left[\begin{array}{ccccc} 1 & 2 & 0 & \frac{1}{2} & 3 \\ 0 & 0 & 1 & -\frac{5}{2} & -2 \end{array}\right]$$

$\uparrow y$ ——— $\uparrow w$ Free vars

so

$x = 3 - 2y - \frac{1}{2} w$

$z = -2 + \frac{5}{2} w$

General Solution

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -2 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + u \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{5}{2} \\ 1 \end{bmatrix}$$

(4) $t, u \in \mathbb{R}$ independent parameters

(could just use y, w instead!

(c)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ 2 & 1 & 2 & 3 \\ 1 & 2 & 1 & 2 \end{array}\right]$$

$R_2 - 2R_1$, $R_3 - R_1$

$$\left.\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 3 & 0 & -1 \\ 0 & 3 & 0 & 0 \end{bmatrix}\right\}$$ Really one could stop here

$R_3 - R_2$, $\frac{1}{3} R_2$

$$\left.\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 0 & -\frac{1}{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}\right\}$$ or here

etc

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow 0 = 1 \;!!$$

The system is inconsistent (no solution) (4)

(OVER)

2 (a) $\begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 0 \end{bmatrix}$

$\downarrow$

$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right]$ The unique solⁿ is $\begin{cases} x = 0 \\ y = 0 \end{cases}$

2

(b) We'd need the second row of $\begin{bmatrix} 2 & 1 & 0 \\ 3 & k & 0 \end{bmatrix}$ to be a multiple (by $\frac{3}{2}$) of the first.

2

$k = \frac{3}{2}$

3

(a) $ABC^{-1} = I$

On the left of both sides:

$$A^{-1}(ABC^{-1}) = A^{-1}I$$

$$(A^{-1}A)BC^{-1} = A^{-1}$$

$$IBC^{-1} = A^{-1}$$

$$BC^{-1} = A^{-1}$$

On the right of both sides

$$(BC^{-1})C = A^{-1}C$$

$$B = A^{-1}C$$

(a) so

$$B = \frac{1}{(-1)} \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$$

3

(b)

$$A^{-1}(3A - 2I)$$

$$= 3(A^{-1}A) - 2A^{-1}$$

$$= 3I - 2A^{-1}$$

$$= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & -1 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 \\ -6 & 7 \end{bmatrix}$$

3