Ass. 8

TOTAL = 12

(1) Fully reduce

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \rightleftarrows R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 1 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 & -1 & 1 \end{array}\right]$$

$(-\frac{1}{2})R_3$, then
$R_1 \to R_1 - R_3$
$R_2 \to R_2 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

Thus

$$C^{-1} = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

3

2 (a) $\vec{x} = C^{-1}\vec{b}$, ie

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1}\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -b_1+b_2+b_3 \\ b_1-b_2+b_3 \\ b_1+b_2-b_3 \end{bmatrix}$$

2 consistent, unique solution for given b_1, b_2, b_3.

2 (b) From (a)

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1+5+2 \\ 1-5+2 \\ 1+5-2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

so

$\alpha = 3, \ \beta = -1, \ \gamma = 2$ 2

3 Reduce

$$\left[\begin{array}{cc|c} 1 & 3 & -11 \\ 2 & 2 & -2 \\ 3 & 1 & 7 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{cc|c} 1 & 3 & -11 \\ 0 & -4 & 20 \\ 0 & -8 & 40 \end{array}\right]$$

$-\frac{1}{4}R_2$, then
$R_3 \to R_3 + 8R_2$
$R_1 \to R_1 - 3R_2$

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -5 \\ 0 & 0 & 0 \end{array}\right]$$

Sol^n: there are unique scalars $x = 4, \ y = -5$. 3

(4) Similarly reduce $\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 2 & 2 & 0 \\ 3 & 1 & 0 \end{array}\right]$

to $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$, thereby

forcing $x = y = 0$. 2

Comment: it is this forcing which indicates $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ are linearly independent.