Ass. 9

TOTAL = 21

(1) Easiest test - the $3\times3$ matrix

$$A = \begin{bmatrix} c & 1 & 0 \\ 1 & c & 1 \\ 0 & 1 & c \end{bmatrix}$$

is invertible ($\det A \neq 0$) just when its columns are linearly independent. So expand along row 1 to get

$$\det(A) = c(c^2-1) - c = c(c^2-2)$$

cols thus dependent when $0 = c(c^2-2)$

Answer: vectors independent except when $c = 0, \sqrt{2}, -\sqrt{2}$. (3)

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(2) Reduce $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 1 & -1 & 2 & 2 \\ 0 & 3 & 3 & -1 \end{array}\right]$

to $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 19/6 \\ 0 & 1 & 0 & 1/6 \\ 0 & 0 & 1 & -1/2 \end{array}\right]$

Yes: $\vec{b}$ is in the span. (2)

$$\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \frac{19}{6}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \frac{1}{6}\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

(3)

(a) $\begin{vmatrix} -\cos(t) & \sin(t) \\ \sin(t) & \cos(t) \end{vmatrix} = -\cos^2 t - \sin^2 t$

$= -1$ (1)

(b) $\begin{vmatrix} 1 & t^3 \\ t^2 & t^5 \end{vmatrix} = t^5 - t^5 = 0$ (1)

(4)

(a) Best to use row 1 or col 1.

$$\begin{vmatrix} 0 & 0 & x \\ 0 & y & z \\ p & q & r \end{vmatrix} = x(+1)\begin{vmatrix} 0 & y \\ p & q \end{vmatrix}$$

$= -p \times y$ (2)

(b) Best to use col 3.

$$\begin{vmatrix} 3 & 4 & 0 & 5 \\ 1 & 0 & 0 & 2 \\ 11 & 12 & -2 & 13 \\ -1 & 1 & 0 & 6 \end{vmatrix} = (-2)(+1)\begin{vmatrix} 3 & 4 & 5 \\ 1 & 0 & 2 \\ -1 & 1 & 6 \end{vmatrix}$$

$= \cdot$

$= -2\{-4(8) - 1(1)\}$

$= 66$ (2)

5. ① each - [5] total

(a) $\begin{vmatrix} a & 10b \\ c & 10d \end{vmatrix} = 10(12) = \underline{120}$

(b) $\begin{vmatrix} a & b \\ -5c & -5d \end{vmatrix} = -5(12) = \underline{-60}$

(c) $\begin{vmatrix} c & d \\ a & b \end{vmatrix} = (-1)(12) = \underline{-12}$

(d) $\begin{vmatrix} a-2c & b-2d \\ c & d \end{vmatrix} = \underline{12}$

(e) $\begin{vmatrix} 3a & 3b \\ 3c & 3d \end{vmatrix} = 3^2 \cdot 12$

$= \underline{108}$

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(6) Singular means determ. is 0. So

$0 = x[x^3 - 18x] - x[0 - x^3]$

$= 2x^4 - 18x^2$

$= 2x^2(x^2 - 9)$ (3)

Ans. singular for $x = 0, 3, -3$.

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(7) Volume is absolute value of the determinant:

$$\rightarrow \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 6 \\ 3 & 0 & -5 \end{vmatrix}$$

$= 3(15) - 5(-5) = 70$

Ans: volume = 70 (2)