DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 1503

Paper Assignment 10

Instructions: Complete each of the following tasks.

A. Read the text, sections 3.2, 3.4, 3.5 and Appendix A on Complex Numbers (pp. 593–605).

B. Try some of the following problems from the text for practice (not to be handed in). It may be a few days or more before we cover all these topics.

Page 262 – True/False questions
Page 263 – 3(a), 4, 6, 10, 13, 15, 19, 20(a), 27, 28, 29, 34

Page 288 – True/False questions
Page 289 – 1, 2, 3(a), 5, 7(d), 8, 9, 11, 15

Page 604 – True/False questions
Page 604 – 1, 2, 3, 4, 57, 11(a), 12

C. **Hand in** the following problems, as instructed in class.

1. Here let
$$A = \begin{bmatrix} -5 & -8 & 2 \\ 3 & 6 & -2 \\ 4 & 4 & 2 \end{bmatrix} \quad , \quad \mathbf{v} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} .$$

   (a) Show that $\mathbf{v}$ is an eigenvector for A and find the corresponding eigenvalue.
   (Careful: this is easy. You can get the eigenvalue almost for free.)

   (b) In fact, $\lambda = 3$ is another eigenvalue for A. Find an eigenvector for $\lambda = 3$.

   Also describe the corresponding eigenspace parametrically.

   (c) Find the third eigenvalue.

more questions→

2. Let $A = \begin{bmatrix} 8 & 18 \\ -3 & -7 \end{bmatrix}$.

   The question: compute in some reasonably compact form A^{100}.

   Hint: the theory of eigenvectors suggests that it will be useful to use another matrix $P = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. Here is what to do.

   First compute $PAP^{-1} = D$. It should look much 'easier' to work with than A itself. Next observe that this means $A = P^{-1}DP$. Then ponder how $A^{100} = (P^{-1}DP)^{100}$ might usefully (and legally) be simplified to give an answer you could actually write down.

3. For the complex numbers $z = 2 + \imath$ and $w = 1 - \imath$, compute:

   (a) zw

   (b) $\frac{z}{w}$

   (c) $\overline{z - 3w}$

   (d) $|-3w|$