DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 1503

Paper Assignment 6

Instructions: Complete each of the following tasks.

A. Read the text, sections 2.1, 2.3, 2.4 and 2.6. Sections 2.2 and 2.5 are optional and we might not do them.

B. Try some of the following problems from the text for practice (not to be handed in). It will be a few days or more before we cover all these topics.

Page 135 – True/False questions

Page 136 – 1(a), 2, 3(a), 6, 12, 13, 15, 23, 27(a, e), 28

Page 161 – True/False questions

Page 162 – 1, 2, 3, 4, 7, 8, 9, 12, 13, 15, 16, 19, 23, 24, 29

C. **Hand in** the following problems, as instructed in class.

1. **Background**. The *period* of a square matrix A is the smallest positive integer p (if there is any such p at all) such that $A^p = I$, where I is the identity with the same dimensions as A. For example,

$$A = A^1 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \text{ but } A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

so this particular A has period 2.

   (a) For the A just above compute A^{509}. (2)

   (b) By trial and error find the period of (2)

$$B = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}.$$

2. Find (and simplify nicely) the inverse of (2)

$$R = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}.$$

more questions →

3. Write an equation for the plane that

   (a) is perpendicular to the line $\mathbf{x}(t) = \langle 5 - 2t, 5 + t, t \rangle$ and passes through the point $(1, 0, 1)$. (2)

   (b) is parallel to the plane $x - 2y + z = 7$ and passes through the point $(1, 2, 2)$. (2)

   Hint: you should be able to do each of these in one simple line of work. Nothing fancy is needed.

4. Determine x and y in this system of simultaneous equations: (2)

$$\begin{cases} \frac{1}{x} - \frac{2}{y} & = & 5 \\ \frac{2}{x} - \frac{5}{y} & = & 4 \end{cases}$$

   Hint: first treat $\frac{1}{x}, \frac{1}{y}$ as chunks without messing with them much.

5. Consider the points $A(1, 0, 1)$, $B(2, 1, 0)$, $C(-1, 0, 0)$ and $D(9, 6, -4)$ in space $\mathbb{R}^3$. Points A and B determine line L_1; points C and D determine line L_2. (4)

   (a) Give line L_1 in parametric form, using the parameter $t \in \mathbb{R}$.

   (b) Give line L_2 in parametric form, but be sure to use a different parameter $u \in \mathbb{R}$.

   (c) Determine whether the two lines intersect or not. If they do, give the point of intersection.

   Warning: as suggested, you should use different parameters t and u for the two lines. Intuitively, this is because points on the two lines could be moving according to different time units, so the parameter isn't likely the same for both.