DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 1503

Paper Assignment 7

Instructions: Complete each of the following tasks.

A. Read the text, sections 2.3, 2.4 and 2.6. Sections 2.2 and 2.5 are optional and we might not do them.

B. Try some of the following problems from the text for practice (not to be handed in). It will be a few days or more before we cover all these topics.

Page 161 – True/False questions

Page 162 – 1, 2, 3, 4, 7, 8, 9, 12, 13, 15, 16, 19, 23, 24, 29

Page 224 – True/False questions

Page 225 – 1(a,c), 2, 3 (a,b,d), 4, 5, 7, 9

C. **Hand in** the following problems, as instructed in class.

1. Solve the following systems by first reducing the augmented matrix to row reduced echelon form (indicate row operations). Then clearly state and check all solutions (if any).

   (a)
   $$\begin{cases} 2x - y + z &= 7 \\ x + y + 2z &= 5 \\ x - 2y - 3z &= -4 \end{cases}$$

   (b)
   $$\begin{cases} x + 2y - z + 3w &= 5 \\ 2x + 4y \quad\ + w &= 6 \end{cases}$$

   (c)
   $$\begin{cases} x - y + z &= 2 \\ 2x + y + 2z &= 3 \\ x + 2y + z &= 2 \end{cases}$$

More questions over →

2. **Backround** A system of m linear equations in the variables $x_1, \dots, x_n$ is called *homogeneous* if all the 'right hand constants' equal 0. This means that the $m \times 1$ constant column $\vec{b} = \vec{0}$ and that the augmented matrix looks something like

$$\left[\begin{array}{ccc|c} * & \dots & * & 0 \\ * & \dots & * & 0 \\ \vdots & & \vdots & \vdots \\ * & \dots & * & 0 \end{array}\right].$$

A homogeneous system will always be consistent, since for sure $x_1 = x_2 = \cdots = x_n = 0$ works.

Answer these questions:

(a) Solve the homogeneous system

$$\begin{cases} 2x + y &= 0 \\ 3x + ky &= 0 \end{cases},$$

in which the constant $k = 2$, in this part. Clearly state your solution.

(b) For which values of the constant k just above would the system have infinitely many solutions? Show relevant calculations.

3. Suppose $A = \begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$.

(a) Find matrix B given that $ABC^{-1} = I$, the 2×2 identity matrix.

(b) Determine the matrix $A^{-1}(3A - 2I)$.