DEPARTMENT OF MATHEMATICS & STATISTICS
MATH 1503

Paper Assignment 8

Instructions: Complete each of the following tasks.

A. Read the text, sections 2.4, 2.6 and 2.9. Sections 2.7 and 2.8 are optional and we might not do them.

B. Try some of the following problems from the text for practice (not to be handed in). It may be a few days or more before we cover all these topics.

Page 161 – True/False questions
Page 162 – 1, 2, 3, 4, 7, 8, 9, 12, 13, 15, 16, 19, 23, 24, 29

Page 181 – True/False questions
Page 181 – 1(a), 3(a), 4, 5, 6(a), 7, 9(a), 10

Page 224 – True/False questions
Page 225 – 1(a,c), 2, 3 (a,b,d), 4, 5, 7, 9

C. **Hand in** the following problems, as instructed in class.

1. Use the row reduction method (text, pp. 216-220) to find the inverse of

$$C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

2. This question uses the matrix C in the previous question. You do not have to repeat the calculations from there. But it is a good idea right now to check that your $C^{-1}C = I$.

   (a) You are given general constants b_1, b_2, b_3. (You aren't allowed to just choose your favorites!) Use the matrix inverse method involving C^{-1} to solve the system

$$\begin{cases} \quad\; y + z = b_1 \\ x \quad\;\; + z = b_2 \\ x + y \quad\;\; = b_3 \end{cases}$$

   Your solution, if any, will depend somehow on b_1, b_2, b_3. Clearly state whether the system is consistent or not.

more questions→

(b) Find scalars α, β, γ such that

$$\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} = \alpha \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \gamma \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Comments (no new problem). The upshot of the previous part is that the left hand column is a linear combination of the the three right hand columns, i.e. it is in the *span* of the three right hand columns. Indeed, more generally we can show that those three right hand columns span all of $\mathbb{R}^3$. By now you should have observed where those right hand columns came from!

3. Find, if possible, scalars x, y such that (3)

$$x \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -11 \\ -2 \\ 7 \end{bmatrix}.$$

Hint: if x, y actually can be found, this says the column vector on the right is in the *span* of the two column vectors on the left. To do this problem, reinterpret it as a system of three equations in two unknowns and try to solve by the usual row reduction method.

4. Show that the equation (3)

$$x \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution $\begin{bmatrix} x \\ y \end{bmatrix}$.

Clearly state this unique solution.

Comment: Since we now know we have a unique solution and since the right-hand vector is the zero vector, we may conclude that the two column vectors on the left are *linearly independent*.