

DEPARTMENT OF MATHEMATICS & STATISTICS  
MATH 1503

Paper Assignment 9

**Instructions:** Complete each of the following tasks.

**A.** Read the text, sections 3.1, 3.2 and 3.4.

**B.** Try some of the following problems from the text for practice (not to be handed in). It may be a few days or more before we cover all these topics.

**Page 224** – True/False questions

**Page 225** – 1(a,c), 2, 3 (a,b,d), 4, 5, 7, 9

**Page 244** – True/False questions

**Page 245** – 1(a,c), 2, 3(a), 7, 9, 12

**Page 262** – True/False questions

**Page 263** – 3(a), 4, 6, 10, 13, 15, 19, 20(a), 27, 28, 29, 34

**Page 288** – True/False questions

**Page 289** – 1, 2, 3(a), 5, 7(d), 8, 9, 11, 15

**C. Hand in** the following problems, as instructed in class.

1. Find all values of the constant  $c$  such that the following set of vectors in  $\mathbb{R}^3$  is linearly independent.

$$\left\{ \begin{bmatrix} c \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ c \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ c \end{bmatrix} \right\}$$

2. Determine whether the vector  $\mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$  is in the span of this set of

vectors:  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

more questions  $\rightarrow$

3. Compute and simplify each of the following determinants:

$$(a) \begin{vmatrix} -\cos(t) & \sin(t) \\ \sin(t) & \cos(t) \end{vmatrix} \quad (b) \begin{vmatrix} 1 & t^3 \\ t^2 & t^5 \end{vmatrix}$$

4. Compute these determinants by expansion along any row or column which seems convenient.

$$(a) \begin{vmatrix} 0 & 0 & x \\ 0 & y & z \\ p & q & r \end{vmatrix}$$

$$(b) \begin{vmatrix} 3 & 4 & 0 & 5 \\ 1 & 0 & 0 & 2 \\ 11 & 12 & -2 & 13 \\ -1 & 1 & 0 & 6 \end{vmatrix}$$

5. Assume in this question that  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 12$ . Now compute

$$(a) \begin{vmatrix} a & 10b \\ c & 10d \end{vmatrix} \quad (b) \begin{vmatrix} a & b \\ -5c & -5d \end{vmatrix} \quad (c) \begin{vmatrix} c & d \\ a & b \end{vmatrix}$$

$$(d) \begin{vmatrix} a - 2c & b - 2d \\ c & d \end{vmatrix} \quad (e) \begin{vmatrix} 3a & 3b \\ 3c & 3d \end{vmatrix}$$

6. For which real numbers  $x$  is the following matrix singular? (A singular matrix is one with no inverse.)

$$\begin{bmatrix} 0 & x & x \\ x & 18 & x^2 \\ x & x^2 & 0 \end{bmatrix}$$

7. Use a determinant to find the volume of the *parallelepiped* (i.e. ‘skewed brick’) spanned by the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 3 \\ 6 \\ -5 \end{bmatrix}$$

in  $\mathbb{R}^3$ . (See problem 12 on page 247 of the text if this topic is unfamiliar to you.)