STUDENT'S NAME: ANSWERS ID #: __________

Department of Mathematics & Statistics

MATH 1503: Mid-Term Test I, October, 2014

Instructions. Do each question. No calculators, notes or other study aids are permitted. Use the reverse side if you need extra space; but **tell us where to look** for such answers.

To save space we often write vectors in row format.

MARKS

Version 1: $v = \langle 3,-1,1\rangle$

2: $v = \langle 4,-1,1\rangle$

3: $v = \langle 5,-1,1\rangle$

TOTAL = 22

1. For the vectors $\mathbf{v} = \langle 3, -1, 1\rangle$ and $\mathbf{w} = \langle -1, 1, 2\rangle$ find:

		Version 1	Version 2	Version 3
(2)	(a) $4\mathbf{v} - 2\mathbf{w}$	$= 4\langle 3,-1,1\rangle - 2\langle -1,1,2\rangle$ $= \underline{\langle 14,-6,0\rangle}$	$4\langle 4,-1,1\rangle$ $-2\langle -1,1,2\rangle$ $= \underline{\langle 18,-6,0\rangle}$	$4\langle 5,-1,1\rangle$ $-2\langle -1,1,2\rangle$ $= \underline{\langle 22,-6,0\rangle}$
(2)	(b) $\mathbf{v}\cdot\mathbf{w}$	$= \underline{-2}$	$= \underline{-3}$	$= \underline{-4}$
(2)	(c) $\mathbf{v}\times\mathbf{w}$	$= \underline{\langle -3,-7,2\rangle}$ $3\ -1\ 1$ $-1\ 1\ 2$	$= \underline{\langle -3,-9,3\rangle}$ $4\ -1\ 1$ $-1\ 1\ 2$	$= \underline{\langle -3,-11,4\rangle}$ $5\ -1\ 1$ $-1\ 1\ 2$
(2)	(d) $\lVert\mathbf{w}\rVert$	$= \underline{\sqrt{6}}$	$\underline{\sqrt{6}}$	$\underline{\sqrt{6}}$

More questions →

2. This question concerns the plane

$$\mathcal{P} : x + 2y - z = 2$$

and the point $D = (1, 2, 9)$ in $\mathbb{R}^3$. $D = (1,2,15)$ $D = (1,2,21)$

Version 1 | Version 2 | Version 3

(3) (a) Find the parametric equations for the line through D which is perpendicular to the plane $\mathcal{P}$. Call this line L.

Version 1:

$$\begin{cases} x = 1 + t \\ y = 2 + 2t \\ z = 9 - t \end{cases} \quad (t \in \mathbb{R})$$

dir^n vect $= \langle 1, 2, -1 \rangle$ for line

(Being perp. to plane that L is, this is also normal vector to plane.)

Version 2:

$$\begin{cases} x = 1 + t \\ y = 2 + 2t \\ z = 15 - t \end{cases} \quad (t \in \mathbb{R})$$

dir^n $= \langle 1, 2, -1 \rangle$

Version 3:

$$\begin{cases} x = 1 + t \\ y = 2 + 2t \\ z = 21 - t \end{cases} \quad (t \in \mathbb{R})$$

dir^n $= \langle 1, 2, -1 \rangle$

(2) (b) Determine the point Q where the line L from part(a) meets the plane $\mathcal{P}$.

Version 1:

$2 = (1+t) + 2(2+2t) - (9-t)$

$2 = -4 + 6t$, so $t = 1$

$Q = (2, 4, 8)$

Version 2:

$2 = (1+t) + 2(2+2t) - (15-t)$

$2 = -10 + 6t$, so $t = 2$

$Q = (3, 6, 13)$

Version 3:

$2 = (1+t) + 2(2+2t) - (21-t)$

$2 = -16 + 6t$, so $t = 3$

$Q = (4, 8, 18)$

(2) (c) Determine the distance from D to the plane $\mathcal{P}$. (Hint: it is the distance between two points already under discussion.)

Version 1:

Same as distance from D to Q

$D = (1, 2, 9)$

$Q = (2, 4, 8)$

dist $= \sqrt{1 + 4 + 1}$

$= \sqrt{6}$

Version 2:

$D = (1, 2, 15)$

$Q = (3, 6, 13)$

dist $= \sqrt{4 + 16 + 4}$

$= \sqrt{24}$

Version 3:

$D = (1, 2, 21)$

$Q = (4, 8, 18)$

dist $= \sqrt{9 + 36 + 9}$

$= \sqrt{54}$

More questions →

(2) 3. Find the projection of $\mathbf{a} = \langle 1, 2, 2 \rangle$ onto $\mathbf{b} = \langle -1, 1, 1 \rangle$.

$$\text{proj}_{\vec{b}} \vec{a} = \frac{\bar{a} \cdot \bar{b}}{\bar{b} \cdot \bar{b}} \vec{b} = \frac{3}{3} \langle -1, 1, 1 \rangle = \underline{\langle -1, 1, 1 \rangle}$$

4. In this question $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$.

(3) (a) Determine scalars x and y such that

$$x\mathbf{u} + y\mathbf{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

or explain why there are no such scalars.

so $x \begin{bmatrix} 1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, giving

$$\begin{cases} x + 3y = 1 \\ 2x + 8y = 1 \end{cases}, \text{ so } \quad \begin{aligned} x &= 1 - 3y \\ 2(1-3y) + 8y &= 1 \\ 2y = -1, \quad y &= -\tfrac{1}{2}; \ x = \tfrac{5}{2} \end{aligned}$$

ANSWER: $x = \frac{5}{2}$

$y = -\frac{1}{2}$

(2) (b) Is the zero vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ a linear combination of $\mathbf{u}$ and $\mathbf{w}$? Explain.

<u>Yes:</u> $\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\vec{u} + 0\vec{w}$, regardless of what vectors $\vec{u}, \vec{w}$ actually are.