**STUDENT'S NAME:**________________________________ **ID #:**______________

DEPARTMENT OF MATHEMATICS & STATISTICS

# MATH 1503

**MID-TERM TEST** **7:00 P.M. – OCTOBER 20, 2005** **TIME: 1.5 HOURS**

**No calculators, notes or other study aids permitted.**

MARKS

1. For the vectors $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 4 \\ -1 \\ 5 \end{bmatrix}$, find:

(1) (a) $4\vec{v} - 2\vec{w}$

(1) (b) $\vec{v} \cdot \vec{w}$

(1) (c) $\vec{v} \times \vec{w}$

(1) (d) $\|\vec{w}\|$

(1) (e) the projection of $\vec{v}$ onto $\vec{w}$.

2. Let $\mathcal{P}_1 : x - 2y + 3z = 4$ and $\mathcal{P}_2 : 2x - 5y - 4z = 1$ be two planes.

(1) (a) Show that $\mathcal{P}_1$ and $\mathcal{P}_2$ are orthogonal (perpendicular).

(2) (b) Find the vector equation of the line of intersection of $\mathcal{P}_1$ and $\mathcal{P}_2$.

(2) (c) Find an equation for the plane through the point $A = (18, 7, 0)$ with normal vector $\vec{n}$ orthogonal to the normals of the planes $\mathcal{P}_1$ and $\mathcal{P}_2$ above.

(2) (d) Find the distance from $\mathcal{P}_1$ to the origin $O = (0, 0, 0)$.

3. Consider the system of linear equations

$$\begin{array}{rrrcl} x & & +\,z & = & 5 \\ -x & -\,y & +\,z & = & 2 \\ & 2y & +\,z & = & 1 \end{array}.$$

(1) (a) Set up the augmented matrix for this system.

(4) (b) Reduce the augmented matrix your obtained in part (a) to row echelon form. Clearly indicate the elementary row operations which you use. (Use the reverse side of this page if you need more space.)

(2) (c) Use your result in part (b) to determine whether the given system has any solu-

4. In this question you are not given the original system, but instead the result of reducing the augmented matrix to echelon form. In each case, indicate whether the system is consistent (has one or more solutions), or inconsistent (has no solution). In the consistent cases, clearly state all solutions in column vector form.

(2) (a) A system in $x, y$ leads to

$$\left[\begin{array}{cc|c} 1 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right].$$

(2) (b) A system in $x, y, z$ leads to

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 3 \end{array}\right].$$

(2) (c) A system in $x, y, z, w$ leads to

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & -1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$