University of New Brunswick **Department of Mathematics & Statistics**

Math 1503 Midterm 2– Review Fall 2011

The problems that was addressed in italic are from "Linear Algebra", fourth edition, Schaum's.

1. Consider the system *[3.6. P. 90]*

$$\begin{aligned} x + ay &= 4 \\ ax + 9y &= b \end{aligned}$$

   (a) For what values of a does the system have a unique solution?

   (b) For what values of a, b the system has more than one solution?

2. Solve each of the following systems: *[3.11 P. 92]* (Solve the system using augmented matrix)

$$\begin{array}{lll} x+2y-4z = -4 & x+2y-3z = -1 & x+2y-3z = 1 \\ 2x+5y-9z = -10 & -3x+y-2z = -7 & 2x+5y-8z = 4 \\ 3x-2y+3z = 11 & 5x+3y-4z = 2 & 3x+8y-13z = 7 \\ (a) & (b) & (c) \end{array}$$

3. Solve the following systems: *[3.12 P. 92]* (Solve using augmented matrix)

$$\begin{array}{ll} x_1-3x_2+2x_3-x_4+2x_5 = 2 & x_1+2x_2-3x_3+4x_4 = 2 \\ 3x_1-9x_2+7x_3-x_4+3x_5 = 7 & 2x_1+5x_2-2x_3+x_4 = 1 \\ 2x_1-6x_2+7x_3+4x_4-5x_5 = 7 & 5x_1+12x_2-7x_3+6x_4 = 3 \\ (a) & (b) \end{array}$$

4. Solve: *[3.22 P. 96]*

$$\begin{array}{lll} x+2y-z = 3 & x-2y+4z = 2 & x+y+3z = 1 \\ x+3y+z = 5 & 2x-3y+5z = 3 & 2x+3y-z = 3 \\ 3x+8y+4z = 17 & 3x-4y+6z = 7 & 5x+7y+z = 7 \\ (a) & (b) & (c) \end{array}$$

5. Solve: *[3.23 P. 97]*

$$\begin{aligned} x_1+2x_2-3x_3-2x_4+4x_5 &= 1 \\ 2x_1+5x_2-8x_3-x_4+6x_5 &= 4 \\ x_1+4x_2-7x_3+5x_4+2x_5 &= 8 \end{aligned}$$

6. The following matrices are the reduced row echelon form of a system of linear equations. Write the general solution; indicate free variables, if any.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & -1 & 9 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ (b) $\left[\begin{array}{ccccc|c} 1 & 0 & 0 & -2 & 0 & 3 \\ 0 & 0 & 1 & 1 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 5 \end{array}\right]$ (c) $\left[\begin{array}{cccc|c} 1 & -2 & 0 & -4 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

7. Find the inverse if possible: *[2.18 P. 46]*

(a) $\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -3 \\ 1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

8. Find the inverse of A: *Example 3.20 p. 85*

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

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(a) $A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$ (b) $B = \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & -6 \end{bmatrix}$

9. Consider the following system of linear equations:

$$\begin{aligned} 5x - 8y &= 10 \\ -2x + 3y &= 7 \end{aligned}$$

Solve the system using inverse of the coefficient matrix: $\mathbf{x} = A^{-1}\mathbf{b}$

10. Find the inverse of: *[3.32, P. 102]*

$$B = \begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & -2 \\ 3 & 13 & -6 \end{bmatrix}$$

11. Write $\mathbf{v}$ as a linear combinations of $\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}$, where: *[3.24 P. 98]*

    (a) $\mathbf{v} =< 3, 10, 7 >$ and $\mathbf{u_1} =< 1, 3, -2 >$, $\mathbf{u_2} =< 1, 4, 2 >$, $\mathbf{u_3} =< 2, 8, 1 >$.

    (b) $\mathbf{v} =< 2, 7, 10 >$ and $\mathbf{u_1} =< 1, 2, 3 >$, $\mathbf{u_2} =< 1, 3, 5 >$, $\mathbf{u_3} =< 1, 5, 9 >$.

    (c) $\mathbf{v} =< 1, 5, 4 >$ and $\mathbf{u_1} =< 1, 3, -2 >$, $\mathbf{u_2} =< 2, 7, -1 >$, $\mathbf{u_3} =< 1, 6, 7 >$.

12. Determine whether $\mathbf{v}$ is a linear combination of the remaining vectors:

    (a) $\mathbf{v} =< 1, 2, 3 >$ and $\mathbf{u_1} =< 1, 3, 0 >$, $\mathbf{u_2} =< 0, 1, -1 >$, $\mathbf{u_3} =< 1, 0, -1 >$.

    (b) $\mathbf{v} =< 2, -4, 1, 2 >$ and $\mathbf{u_1} =< 1, 2, 3, -1 >$, $\mathbf{u_2} =< 1, 3, 1, 1 >$.

13. Consider $A = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix}$

    (a) Use Gauss-Jordan method to find A^{-1}.

    (b) Using A^{-1} from part (a), solve the following system:

    $$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 1 & 2 \\ -2 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$$

14. Determine whether the following set of vectors are linearly independent. If they are dependent, write one as a linear combination of the others.

    (a) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \right\}$, (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

15. For each set of vectors find all values of the constant a such that the set is linearly independent.

    (a) $\left\{ \begin{bmatrix} a \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} \right\}$, (b) $\left\{ \begin{bmatrix} a \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ a \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ a \end{bmatrix} \right\}$

16. Which of the following sets of vectors span $\mathbb{R}^3$?

    (a) $\left\{ \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 7 \\ 3 \end{bmatrix} \right\}$

    (b) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} \right\}$

17. Determine if the vector $\mathbf{v} = [3, 2, -1]$ is in the span of the following set of vectors:

    $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$