STUDENT'S NAME: ANSWERS. ID #:

Department of Mathematics & Statistics

MATH 1503: Mid-Term Test II, November, 2014

Instructions. Do each question. No calculators, notes or other study aids are permitted. Use the reverse side if you need extra space; but **tell us where to look** for such answers.

MARKS
TOTAL = 24

1. Consider the system of linear equations

$$\begin{array}{rrrcr} x & & +2z & = & 2 \\ -x & -y & +z & = & 1 \\ & -2y & +z & = & -4 \end{array}.$$

(6) (a) Give the augmented matrix for this system. Then reduce the augmented matrix to row reduced echelon form. Clearly indicate the elementary row operations which you use. (Use the reverse side of this page if you need more space.)

Version 1

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ -1 & -1 & 1 & 1 \\ 0 & -2 & 1 & -4 \end{array}\right]$$

$R_2 \to R_2 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & -1 & 3 & 3 \\ 0 & -2 & 1 & -4 \end{array}\right]$$

$(-1)R_2$, then $R_3 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & -5 & -10 \end{array}\right]$$

$-\frac{1}{5}R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 - 2R_3$
$R_2 + 3R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

SAME ERO's →

Version 2

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & -1 & 1 & 2 \\ 0 & -2 & 1 & 11 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & -1 & 3 & 3 \\ 0 & -2 & 1 & 11 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & -5 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

(b) Use your result in part (a) to determine whether the given system has any solutions; clearly describe the solution or solutions (if any).

Unique solution:

$$\begin{cases} x = -2 \\ y = 3 \\ z = 2 \end{cases}$$

Unique solution:

$$\begin{cases} x = 3 \\ y = -6 \\ z = -1 \end{cases}$$

2. In this question you are not given the original system, but instead the result of reducing the augmented matrix to row echelon form. In each case, clearly explain whether the system is consistent (has one or more solutions), or inconsistent (has no solution). In the consistent cases, clearly describe all solutions.

(2) (a) A system in x_1, x_2, x_3, x_4 leads to

$$\left[\begin{array}{cccc|c} 1 & -1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Version 1:

$x_1 - x_2 + 2x_4 = 1$

$x_3 + 2x_4 = 5$

so $\begin{cases} x_1 = 1 + x_2 - 2x_4 \\ x_2 = x_2 \quad \text{(free)} \\ x_3 = 5 - 2x_4 \\ x_4 = x_4 \quad \text{(free)} \end{cases}$

OR $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 5 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$ (inf. many sol'ns)

Version 2:

$$\left[\begin{array}{ccccc} 1 & -1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_1 - x_2 + 2x_4 = 1$

$x_3 + 2x_4 = 3$

so $\begin{cases} x_1 = 1 + x_2 - 2x_4 \\ x_2 = x_2 \quad \text{(free)} \\ x_3 = 3 - 2x_4 \\ x_4 = x_4 \quad \text{(free)} \end{cases}$

or $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

(inf. many sol'ns)

(2) (b) A system in x, y, z leads to

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right].$$

Inconsistent - no solution (last row indicates $0 = 1$)

(2) (c) A system in x, y, z leads to

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

Unique solution

$\begin{cases} x = 2 \\ y = -5 \\ z = 1 \end{cases}$

Version 2

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Unique Solution.

$\begin{cases} x = 2 \\ y = 5 \\ z = 1 \end{cases}$

3. Carry out the indicated computations, if possible, using the following matrices P and Q. When not possible, you must indicate why.

$$P = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} \quad , \quad Q = \begin{bmatrix} 2 & 4 & -1 \\ 2 & -3 & 1 \end{bmatrix} \quad .$$

(2) (a) Compute, if possible, PQ.

$$PQ = \begin{bmatrix} 4 & 1 & 0 \\ 14 & 0 & 1 \end{bmatrix}$$

(2) (b) Compute, if possible, QP. – impossible – mismatched inner dimensions

(2) (c) Compute, if possible, P^{-1}. $P^{-1} = \frac{1}{1} \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -3 & 1 \end{bmatrix}$

(2) (d) Compute, if possible, $Q - 2P$. – impossible – Q and $2P$ have different dimensions.

(4) 4. Find the inverse of the matrix $C = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$. Show your work.

Fully reduce $\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$R_1 \rightleftarrows R_2$ $\left[\begin{array}{cccccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$

$R_3 - 2R_1$ $\left[\begin{array}{cccccc} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & 0 & -2 & 1 \end{array}\right]$

$R_1 \to R_1 - R_2$
$R_3 \to R_3 + 2R_2$ $\left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$

$R_2 \to R_2 - R_3$ $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 2 & -1 \\ 0 & 0 & 1 & 2 & -2 & 1 \end{array}\right]$

$\underbrace{}_{I_3}$

Thus $C^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 2 & -1 \\ 2 & -2 & 1 \end{bmatrix}$